

## Section 12.2 part 1

$F \subseteq K$  - field extension       $\text{Gal}_F K = \{ \sigma : K \rightarrow K \mid \sigma(c) = c \text{ for every } c \in F \}$

field isomorphism  
automorphism

$\sigma|_F = \text{identity map}$

Intermediate fields

$$F \subseteq E \subseteq K$$

### Galois Correspondence

Subgroups

$$\text{Gal}_E K \subseteq \text{Gal}_F K$$

$$E_H = \{ \sigma \in K \mid \sigma(\xi) = \xi \text{ for every } \xi \in H \}$$

fixed field of  $H$

$$H \subseteq \text{Gal}_F K$$

### Extreme cases

$$E = K$$



$$\langle \iota \rangle = \text{Gal}_K K \subseteq \text{Gal}_F K$$

$$E = F$$

$$\text{Gal}_F K \subseteq \text{Gal}_F K$$

$$F = \mathbb{Q} \quad K = \mathbb{Q}(\sqrt[3]{2})$$

$$\text{Gal}_{\mathbb{Q}} \mathbb{Q}(\sqrt[3]{2}) = \langle \iota \rangle = \text{Gal } \mathbb{Q}(\sqrt[3]{2}) / \mathbb{Q}(\sqrt[3]{2})$$

it may happen that  $\text{Gal}_F K$  fixes a subfield  $E > F$ .

$$\mathbb{Q}(\sqrt[3]{2})$$

In general, what may go wrong?

$$E \rightsquigarrow H = \text{Gal}_E K \rightsquigarrow E_H$$

Clearly,  $E_H \supseteq E$

Is every intermediate subfield  
the fixed field of some subgroup

However, elements of  $\text{Gal}_E K$   
may fix a bigger subfield besides  $E$   
 $E_H \neq E$

of  $\text{Gal}_F K$ ? - NO

Under which extra conditions the answer is YES? (Th 12.9)

$$\text{Gal}_F K \supseteq H \Leftrightarrow E_H \text{ is } \text{Gal}_{E_H}^K$$

Is every subgroup  $H \subseteq \text{Gal}_F K$  the Galois group of an intermediate field (Th 12.8)

$$\text{Clearly, } H \subseteq \text{Gal}_{E_H}^K$$

However, there may be elements of  $\text{Gal}_{E_H}^K$  are not in  $H$  but still fix  $E_H$